

פרק 1 - פונקציות טריגונומטריות - פתרון

1) המר את הזוויות הבאות ממעלות לרדיאנים וההיפך:

$$1)\alpha = \frac{30^\circ \cdot \pi}{180^\circ} = \frac{\pi}{6}$$

$$2)\alpha = \frac{90^\circ \cdot \pi}{180^\circ} = \frac{\pi}{2}$$

$$3)\alpha = \frac{45^\circ \cdot \pi}{180^\circ} = \frac{\pi}{4}$$

$$4)\alpha = \frac{350^\circ \cdot \pi}{180^\circ} = \frac{35\pi}{18}$$

$$5)\alpha = \frac{225^\circ \cdot \pi}{180^\circ} = \frac{5\pi}{4}$$

$$6)\alpha = \frac{-60^\circ \cdot \pi}{180^\circ} = \frac{-\pi}{3}$$

$$7)\alpha = \frac{-270^\circ \cdot \pi}{180^\circ} = \frac{-3\pi}{2}$$

$$8)\alpha = \frac{180^\circ \cdot \pi}{180^\circ} = \pi$$

$$9)\alpha = \frac{57\gamma 3^\circ \cdot \pi}{180^\circ} = 0.318\pi$$

$$10)\alpha = \frac{\frac{\pi}{2} \cdot 180}{\pi} = 90^\circ$$

$$11)\alpha = \frac{\frac{\pi}{3} \cdot 180}{\pi} = 60^\circ$$

$$12)\alpha = \frac{\frac{5\pi}{2} \cdot 180}{\pi} = 450^\circ$$

$$13)\alpha = \frac{\frac{3\pi}{2} \cdot 180}{\pi} = 270^\circ$$

$$14)\alpha = \frac{-3\pi \cdot 180}{\pi} = -540^\circ$$

$$15)\alpha = \frac{-\frac{\pi}{10} \cdot 180}{\pi} = -18^\circ$$

(2) השתמש בזהויות הטריגונומטריות והוכח את הקשרים הבאים:

$$1. \tan(180 - \alpha) \cos(180 - \alpha) + \sin(-\alpha) = 0$$

$$\left\{ \tan(180 - \alpha) = \frac{\sin(180 - \alpha)}{\cos(180 - \alpha)}, \cos(180 - \alpha) = -\cos(\alpha), \sin(-\alpha) = -\sin(\alpha), \sin(180 - \alpha) = \sin(\alpha) \right\}$$

$$\frac{\sin(\alpha)}{-\cos(\alpha)} \cdot (-\cos(\alpha)) - \sin(\alpha) = 0$$

$$\sin(\alpha) - \sin(\alpha) = 0$$

$$2. \frac{\sin(180 - \alpha)}{1 - \cos(-\alpha)} = \frac{1 - \cos(180 - \alpha)}{\cos(90 - \alpha)}$$

$$\left\{ \cos(180 - \alpha) = -\cos(\alpha), \sin(180 - \alpha) = \sin(\alpha), \cos(-\alpha) = \cos(\alpha), \cos(90 - \alpha) = \sin(\alpha) \right\}$$

$$\frac{\sin(\alpha)}{1 - \cos(\alpha)} = \frac{1 - (-\cos(\alpha))}{\sin(\alpha)}$$

$$\sin^2(\alpha) = (1 - \cos(\alpha)) \cdot (1 + \cos(\alpha))$$

$$\sin^2(\alpha) = 1 - \cos^2(\alpha)$$

$$1 = \sin^2(\alpha) + \cos^2(\alpha)$$

$$3. 1 - 2 \sin^2 \frac{\alpha}{2} = \cos \alpha$$

$$\left\{ \sin^2(\alpha) = \frac{1}{2}(1 - \cos(2\alpha)) \Rightarrow \sin^2\left(\frac{\alpha}{2}\right) = \frac{1}{2}(1 - \cos(\alpha)) \right\}$$

$$1 - 2 \cdot \left(\frac{1}{2}(1 - \cos(\alpha))\right) = \cos(\alpha)$$

$$1 - 1 + \cos(\alpha) = \cos(\alpha)$$

$$0 = 0$$

$$4. (\sin \alpha + \cos \alpha)^2 = 1 + \sin(2\alpha)$$

$$\sin^2(\alpha) + 2 \cdot \sin(\alpha) \cdot \cos(\alpha) + \cos^2(\alpha) = 1 + \sin(2\alpha)$$

$$\left\{ \sin^2(\alpha) + \cos^2(\alpha) = 1, 2 \cdot \sin(\alpha) \cdot \cos(\alpha) = \sin(2\alpha) \right\}$$

$$1 + \sin(2\alpha) = 1 + \sin(2\alpha)$$

$$5. \frac{\cos \alpha - \cos^3 \alpha}{\sin \alpha - \sin^3 \alpha} = \tan \alpha$$

$$\frac{\cos \alpha(1 - \cos^2 \alpha)}{\sin \alpha(1 - \sin^2 \alpha)} = \frac{\sin \alpha}{\cos \alpha}$$

$$\left\{ 1 - \cos^2 \alpha = \sin^2(\alpha), 1 - \sin^2 \alpha = \cos^2(\alpha) \right\}$$

$$\frac{\cos \alpha \cdot \sin^2 \alpha}{\sin \alpha \cdot \cos^2 \alpha} = \frac{\sin \alpha}{\cos \alpha}$$

$$\frac{\sin \alpha}{\cos \alpha} = \frac{\sin \alpha}{\cos \alpha}$$

$$6. \frac{1 - \cos(2\alpha)}{\sin(2\alpha)} \cdot \frac{\cos(\alpha)}{1 - \cos(\alpha)} = \cot\left(\frac{\alpha}{2}\right)$$

$$\left\{ \begin{array}{l} 1 - \cos(2\alpha) = 2 \sin^2(\alpha), 2 \cdot \sin(\alpha) \cdot \cos(\alpha) = \sin(2\alpha), \cot\left(\frac{\alpha}{2}\right) = \frac{\cos\left(\frac{\alpha}{2}\right)}{\sin\left(\frac{\alpha}{2}\right)} \end{array} \right\}$$

$$\frac{2 \sin^2(\alpha)}{2 \cdot \sin(\alpha) \cdot \cos(\alpha)} \cdot \frac{\cos(\alpha)}{1 - \cos(\alpha)} = \frac{\cos\left(\frac{\alpha}{2}\right)}{\sin\left(\frac{\alpha}{2}\right)}$$

$$\left\{ \sin^2\left(\frac{\alpha}{2}\right) = \frac{1}{2}(1 - \cos(\alpha)) \Rightarrow 1 - \cos(\alpha) = 2 \sin^2\left(\frac{\alpha}{2}\right) \right\}$$

$$\sin(\alpha) \cdot \frac{1}{2 \sin^2\left(\frac{\alpha}{2}\right)} = \frac{\cos\left(\frac{\alpha}{2}\right)}{\sin\left(\frac{\alpha}{2}\right)}$$

$$\left\{ \sin(\alpha) = 2 \cdot \sin\left(\frac{\alpha}{2}\right) \cdot \cos\left(\frac{\alpha}{2}\right) \right\}$$

$$\frac{2 \cdot \sin\left(\frac{\alpha}{2}\right) \cdot \cos\left(\frac{\alpha}{2}\right)}{2 \sin^2\left(\frac{\alpha}{2}\right)} = \frac{\cos\left(\frac{\alpha}{2}\right)}{\sin\left(\frac{\alpha}{2}\right)}$$

$$\frac{\cos\left(\frac{\alpha}{2}\right)}{\sin\left(\frac{\alpha}{2}\right)} = \frac{\cos\left(\frac{\alpha}{2}\right)}{\sin\left(\frac{\alpha}{2}\right)}$$

$$7. \cos^4 \alpha - \sin^4 \alpha = 1 - 2 \sin^2 \alpha$$

$$\cos^2 \alpha \cdot \cos^2 \alpha - \sin^2 \alpha \cdot \sin^2 \alpha = 1 - 2 \sin^2 \alpha$$

$$(1 - \sin^2 \alpha) \cdot \cos^2 \alpha - (1 - \cos^2 \alpha) \cdot \sin^2 \alpha = 1 - 2 \sin^2 \alpha$$

$$\cos^2 \alpha - \cos^2 \alpha \cdot \sin^2 \alpha - \sin^2 \alpha + \cos^2 \alpha \cdot \sin^2 \alpha = 1 - 2 \sin^2 \alpha$$

$$\cos^2 \alpha - \sin^2 \alpha = 1 - 2 \sin^2 \alpha$$

$$\cos^2 \alpha - \sin^2 \alpha = 1 - 2(1 - \cos^2 \alpha)$$

$$\left\{ \cos^2 \alpha - \sin^2 \alpha = \cos(2\alpha) \right\}$$

$$\cos(2\alpha) = 2 \cos^2 \alpha - 1$$

$$8. \cos^2(90 - \alpha) \cot^2 \alpha = 1 - \sin^2 \alpha$$

$$\{\cos(90 - \alpha) = \sin(\alpha)\}$$

$$\sin^2(\alpha) \frac{\cos^2 \alpha}{\sin^2 \alpha} = 1 - \sin^2 \alpha$$

$$\cos^2 \alpha = 1 - \sin^2 \alpha$$

$$1 = \cos^2 \alpha + \sin^2 \alpha$$

$$9. (\tan \alpha + \cot \alpha)^2 - (\tan \alpha - \cot \alpha)^2 = 4$$

$$\left(\frac{\cos \alpha}{\sin \alpha} + \frac{\sin \alpha}{\cos \alpha}\right)^2 - \left(\frac{\sin \alpha}{\cos \alpha} - \frac{\cos \alpha}{\sin \alpha}\right)^2 = 4$$

$$\left(\frac{\cos^2 \alpha + \sin^2 \alpha}{\cos \alpha \sin \alpha}\right)^2 - \left(\frac{\sin^2 \alpha - \cos^2 \alpha}{\cos \alpha \sin \alpha}\right)^2 = 4$$

$$(\cos^2 \alpha + \sin^2 \alpha)^2 - (\sin^2 \alpha - \cos^2 \alpha)^2 = 4(\cos \alpha \sin \alpha)^2$$

$$1 - \sin^4 \alpha - \cos^4 \alpha + 2\cos^2 \alpha \sin^2 \alpha = 4\cos^2 \alpha \sin^2 \alpha$$

$$1 = \sin^4 \alpha + \cos^4 \alpha + 2\cos^2 \alpha \sin^2 \alpha$$

$$1 = (\sin^2 \alpha + \cos^2 \alpha)^2$$

$$1 = 1^2 = 1$$

$$10. \frac{\cos^3 \alpha}{\sin \alpha} - \frac{\sin^3 \alpha}{\cos \alpha} = \frac{(\sin \alpha + \cos \alpha)(\cos \alpha - \sin \alpha)}{\sin \alpha \cos \alpha}$$

$$\frac{\cos^4 \alpha - \sin^4 \alpha}{\sin \alpha \cos \alpha} = \frac{\sin \alpha \cos \alpha - \sin^2 \alpha + \cos^2 \alpha - \sin \alpha \cos \alpha}{\sin \alpha \cos \alpha}$$

$$(\sin^2 \alpha + \cos^2 \alpha) \cdot (\cos^2 \alpha - \sin^2 \alpha) = \cos^2 \alpha - \sin^2 \alpha$$

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$11. \frac{\cos^2 \alpha \sin(90 - \alpha)}{\sin^2 \alpha} = \frac{\cos \alpha \cot \alpha}{\cot(90 - \alpha)}$$

$$\frac{\cos^2 \alpha \cos \alpha}{\sin^2 \alpha} = \frac{\cos \alpha \frac{\cos \alpha}{\sin \alpha}}{\frac{\sin \alpha}{\cos \alpha}}$$

$$\frac{\cos^3 \alpha}{\sin^2 \alpha} = \frac{\cos^3 \alpha}{\sin^2 \alpha}$$

$$12. \frac{2}{\tan(2\alpha)} = \cot(\alpha) - \tan(\alpha)$$

$$\frac{2}{\sin 2\alpha} = \frac{\cos \alpha}{\sin \alpha} - \frac{\sin \alpha}{\cos \alpha}$$

$$\cos 2\alpha$$

$$\frac{2 \cos 2\alpha}{2 \sin \alpha \cos \alpha} = \frac{\cos^2 \alpha - \sin^2 \alpha}{\sin \alpha \cos \alpha}$$

$$2 \cos^2 \alpha - 1 = \cos^2 \alpha - \sin^2 \alpha$$

$$1 = \cos^2 \alpha + \sin^2 \alpha$$