פרק 2- הנגזרת- פתרון

נגזרות בסיסיות וכללי גזירה

גזור את הפונקציות הבאות:

$1. y = e^{\sin x}$

$y' = e^{\sin x} \cdot (\sin x)' = e^{\sin x} \cdot \cos x$

$2. y = mR\cos t(\cos t - \sin t)$

$m, R - const$

$y' = mR[-\sin t(\cos t - \sin t) + \cos t(\cos t - \sin t)'] = mR[-\sin t(\cos t - \sin t) + \cos t(-\sin t - \cos t)] =$

$mR[-2\sin t\cos t + \sin^2 t - \cos^2 t] = mR(-\cos 2t - \sin 2t)$

$3. y = \ln(e^x \cdot x^2)$

$y' = \frac{(e^x \cdot x^2)'}{e^x \cdot x^2} = \frac{e^x \cdot 2x + e^x \cdot x^2}{e^x \cdot x^2} = \frac{2}{x} + 1$

$4. y = 3x^2 + 5x + 2$

$y' = 6x + 5$

$5. y = e^{2x}$

$y' = e^{2x} \cdot (2x)' = 2e^{2x}$

$6. y = \frac{1}{x}$

$y' = -\frac{1}{x^2}$

$7. y = x\ln x$

$y' = 1 \cdot \ln x + x \cdot \frac{1}{x} = \ln x + 1$

$8. y = x^3 \sin x$

$y' = 3x^2 \cdot \sin x + x^3 \cdot (\sin x)' = 3x^2 \cdot \sin x + x^3 \cdot \cos x$

$9. y = 3 + 10x + 0.5\sin(2x)$

$y' = 10 + 0.5 \cdot \cos(2x) \cdot (2x)' = 10 + 0.5 \cdot 2 \cdot \cos(2x) = 10 + \cos(2x)$

$10. y = e^x\sqrt{x^2 + 2}$

$y' = e^x \cdot (\sqrt{x^2 + 2})' + e^x\sqrt{x^2 + 2} = e^x \cdot \frac{(x^2 + 2)'}{2\sqrt{x^2 + 2}} + e^x\sqrt{x^2 + 2} = e^x\sqrt{x^2 + 2} + e^x \cdot \frac{2x}{2\sqrt{x^2 + 2}}$

$11. y = \sqrt{\sin^2 x + \ln x}$

$$y' = \frac{(\sin^2 x + \ln x)'}{2\sqrt{\sin^2 x + \ln x}} = \frac{2\sin x \cos x + \frac{1}{x}}{2\sqrt{\sin^2 x + \ln x}}$$

$12. y = (6x^4 + 7x^3 + x)^{-\frac{1}{2}}$

$$y' = -\frac{1}{2}(6x^4 + 7x^3 + x)^{-\frac{3}{2}}(6x^4 + 7x^3 + x)' = -\frac{1}{2}(6x^4 + 7x^3 + x)^{-\frac{3}{2}}(24x^3 + 21x^2 + 1)$$

$13. y = 2\cos(3t) + 4\sin(3t)$

$$y' = -2\sin(3t)\cdot 3 + 4\cos(3t)\cdot 3 = 12\cos(3t) - 6\sin(3t)$$

$$14. y = \cos x(\frac{1}{x^3 + 3x^2 + 7})$$

$$y' = -\sin x \cdot (\frac{1}{x^3 + 3x^2 + 7}) + \cos x \cdot (\frac{1}{x^3 + 3x^2 + 7})' = \cos x \cdot (-\frac{(x^3 + 3x^2 + 7)'}{(x^3 + 3x^2 + 7)^2}) - \frac{\sin x}{x^3 + 3x^2 + 7} =$$

$$-\cos x \cdot \frac{3x^2 + 6x}{(x^3 + 3x^2 + 7)^2} - \frac{\sin x}{x^3 + 3x^2 + 7}$$

$15. y = (x^2 + 3x - 4)^{20}$

$$y' = 20(x^2 + 3x - 4)^{19} \cdot (x^2 + 3x - 4)' = 20(x^2 + 3x - 4)^{19} \cdot (2x + 3)$$

$$16. y = \sqrt[3]{\frac{x^2 + 1}{1 - x}}$$

$$y' = \frac{1}{3}(\frac{x^2 + 1}{1 - x})^{-\frac{2}{3}} \cdot (\frac{x^2 + 1}{1 - x})' = \frac{1}{3}(\frac{x^2 + 1}{1 - x})^{-\frac{2}{3}} \cdot (\frac{2x(1 - x) - (-1)(x^2 + 1)}{(1 - x)^2}) = \frac{1}{3}(\frac{x^2 + 1}{1 - x})^{-\frac{2}{3}}(\frac{2x - 2x^2 + x^2 + 1}{(1 - x)^2}) =$$

$$\frac{1}{3}(\frac{x^2 + 1}{1 - x})^{-\frac{2}{3}}(\frac{2x - x^2 + 1}{(1 - x)^2})$$

$17. y = \ln(x^4 + 3x^3 + x)$

$$y' = \frac{(x^4 + 3x^3 + x)'}{x^4 + 3x^3 + x} = \frac{4x^3 + 9x^2 + 1}{x^4 + 3x^3 + x}$$

18. $y = B\sin(50x+2)$

$B - const$

$y' = B\cos(50x+2)\cdot 50$

19. $y = \sqrt[3]{x+\sqrt{x}}$

$y' = \frac{1}{3}\cdot(x+\sqrt{x})^{-\frac{2}{3}}(x+\sqrt{x})' = \frac{1}{3}\cdot(x+\sqrt{x})^{-\frac{2}{3}}(1+\frac{1}{2\sqrt{x}})$

20. $y = A\cos(\omega t)+B\sin(\omega t)$

$A, B, \omega - const$

$y' = -A\sin(\omega t)\omega + B\cos(\omega t)\omega$

21. $y = \frac{\tan x}{x^3}$

$y' = \frac{\frac{1}{\cos^2 x}x^3 - 3x^2\tan x}{x^6} = \frac{1}{x^3\cos^2 x} - \frac{3\tan x}{x^4}$

22. $y = \cos(\tan(x))$

$y' = -\sin(\tan x)\cdot(\tan x)' = -\sin(\tan x)\frac{1}{\cos^2 x}$

23. $y = \frac{x}{(5x^4-10x)e^x}$

$\left\{ y = \frac{1}{(5x^3-10)e^x} \right\}$

$y' = -\frac{((5x^3-10)e^x)'}{((5x^3-10)e^x)^2} = -\frac{e^x(15x^2)+e^x(5x^3-10)}{((5x^3-10)e^x)^2} = -\frac{15x^2+5x^3-10}{(5x^3-10)^2 e^x}$

שאלת אתגר:

$$y=\ln(\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}-x})$$

$$y'=\frac{(\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}-x})'}{(\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}-x})}=\frac{\frac{(\sqrt{x^2+1}-x)(\frac{2x}{2\sqrt{x^2+1}}+1)-(\frac{2x}{2\sqrt{x^2+1}}-1)(\sqrt{x^2+1}+x)}{(\sqrt{x^2+1}-x)^2}}{(\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}-x})}=$$

$$\frac{x-\frac{x^2}{\sqrt{x^2+1}}+(\sqrt{x^2+1}-x)-\frac{x^2}{\sqrt{x^2+1}}-x+(\sqrt{x^2+1}+x)}{(\sqrt{x^2+1}-x)(\sqrt{x^2+1}+x)}=$$

$$\frac{-2\frac{x^2}{\sqrt{x^2+1}}+2\sqrt{x^2+1}}{x^2+1-x^2}=\frac{-2x^2+2x^2+1}{\sqrt{x^2+1}}=\frac{1}{\sqrt{x^2+1}}$$

נגזרות נוספות

גזור את הפונקציות הבאות:

$$1.y=x\cdot arctg(x^2)$$

$$y'=1\cdot arctg(x^2)+x\frac{(x^2)'}{1+(x^2)^2}=arctg(x^2)+\frac{2x^2}{1+x^4}$$

$$2.y=arctg(\frac{1+x}{1-x})$$

$$y'=\frac{(\frac{1+x}{1-x})'}{1+(\frac{1+x}{1-x})^2}=\frac{\frac{1\cdot(1-x)-(-1)\cdot(1+x)}{(1-x)^2}}{1+(\frac{1+x}{1-x})^2}=\frac{1-x+1+x}{(1-x)^2+(1+x)^2}=\frac{2}{1-2x+x^2+1+2x+x^2}=\frac{1}{1+x^2}$$

$$3.y=\ln(\arcsin x)$$

$$y'=\frac{(\arcsin x)'}{\arcsin x}=\frac{\frac{1}{\sqrt{1-x^2}}}{\arcsin x}$$

$$4.y=\arcsin(\frac{x}{\sqrt{1+x^2}})$$

$$y'=\frac{(\frac{x}{\sqrt{1+x^2}})'}{\sqrt{1-(\frac{x}{\sqrt{1+x^2}})^2}}=\frac{\frac{1\cdot\sqrt{1+x^2}-\frac{2x^2}{2\sqrt{1+x^2}}}{1+x^2}}{\sqrt{\frac{1+x^2-x^2}{1+x^2}}}=\frac{\frac{1}{\sqrt{1+x^2}}-\frac{x^2}{(1+x^2)^{\frac{3}{2}}}}{\sqrt{\frac{1}{1+x^2}}}=1-\frac{x^2}{(1+x^2)}=\frac{1}{1+x^2}$$

5. $y = \log_5(x)$

$y' = \frac{1}{x}\log_5 e$

6. $y = 6^{x^2}$

$y' = 6^{x^2}\ln 6 \cdot 2x$

7. $y = \arccos(2^x)$

$y' = -\frac{1}{\sqrt{1-(2^x)^2}}(2^x)' = -\frac{2^x \ln 2}{\sqrt{1-2^{2x}}}$

8. $y = 3^{\ln x}$

$y' = 3^{\ln x}\ln 3 \cdot \frac{1}{x}$

9. $y = \log_{10}(x^2 + 5x)$

$y' = \frac{1}{x^2+5x}\log_{10} e(x^2+5x)' = \frac{2x+5}{x^2+5x}\log_{10} e$