

פרק 5- פיתוח פונקציה לטור

פתחו לטור מקלורן את הפונקציות הבאות, עד לאיבר הרביעי:

$$1. f(x) = \tan x$$

$$f'(x) = \frac{1}{\cos^2 x}$$

$$f''(x) = \frac{2 \cos x \sin x}{\cos^4 x} = \frac{2 \sin x}{\cos^3 x}$$

$$f'''(x) = \frac{2 \cos^4 x + 3 \cos^2 x \cdot 2 \sin^2 x}{\cos^6 x}$$

$$f^{(4)}(x) = \frac{4 \sin x}{\cos^3 x} + 6 \left(\frac{2 \sin x \cos^5 x + 4 \cos^3 x \sin^3 x}{\cos^8 x} \right)$$

$$f(0) = 0, f'(0) = 1, f''(0) = 0, f'''(0) = 2, f^{(4)}(0) = 0$$

$$\tan x = x + 2 \frac{x^3}{3!} + \dots = x + \frac{x^3}{3} + \dots$$

$$2. f(x) = e^{-x}$$

$$f'(x) = -e^{-x}$$

$$f''(x) = e^{-x}$$

$$f'''(x) = -e^{-x}$$

$$f^{(4)}(x) = e^{-x}$$

$$f(0) = 1, f'(0) = -1, f''(0) = 1, f'''(0) = -1, f^{(4)}(0) = 1$$

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$

$$3. f(x) = \sqrt{1+x}$$

$$f'(x) = \frac{1}{2\sqrt{1+x}}$$

$$f''(x) = \frac{-\frac{1}{\sqrt{1+x}}}{4(1+x)} = -\frac{1}{4(1+x)^{\frac{3}{2}}}$$

$$f'''(x) = \frac{3}{8} \frac{1}{(1+x)^{\frac{5}{2}}}$$

$$f^{(4)}(x) = -\frac{15}{16} \frac{1}{(1+x)^{\frac{7}{2}}}$$

$$f(0) = 1, f'(0) = \frac{1}{2}, f''(0) = -\frac{1}{4}, f'''(0) = \frac{3}{8}, f^{(4)}(0) = -\frac{15}{16}$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{4 \cdot 2!} + \frac{3x^3}{8 \cdot 3!} - \frac{15x^4}{16 \cdot 4!} + \dots = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \frac{5x^4}{128} + \dots$$

$$4. f(x) = \ln(1+x)$$

$$f'(x) = \frac{1}{(1+x)}$$

$$f''(x) = -\frac{1}{(1+x)^2}$$

$$f'''(x) = \frac{2}{(1+x)^3}$$

$$f^{(4)}(x) = \frac{-6}{(1+x)^4}$$

$$f(0) = 0, f'(0) = 1, f''(0) = -1, f'''(0) = 2, f^{(4)}(0) = -6$$

$$\ln(1+x) = x - \frac{x^2}{2!} + \frac{2x^3}{3!} - \frac{6x^4}{4!} + \dots = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

$$5. f(x) = \ln\left(\frac{1+x}{1-x}\right) = \ln(1+x) - \ln(1-x)$$

$$f'(x) = \frac{1}{1+x} - \frac{-1}{1-x} = \frac{2}{1-x^2}$$

$$f''(x) = \frac{4x}{(1-x^2)^2}$$

$$f'''(x) = \frac{4(1-x^2)^2 + 16x^2(1-x^2)}{(1-x^2)^4}$$

$$f^{(4)}(x) = \frac{16x(1-x^2)}{(1-x^2)^4} + \frac{32x(1-x^2)^3 + 96x^3(1-x^2)^2}{(1-x^2)^6}$$

$$f(0) = 0, f'(0) = 2, f''(0) = 0, f'''(0) = 4, f^{(4)}(0) = 0$$

$$\ln\left(\frac{1+x}{1-x}\right) = 2x + 4\frac{x^3}{3!} + \dots = 2x + \frac{2x^3}{3} + \dots$$

$$6. f(x) = \frac{1}{1-x}$$

$$f'(x) = \frac{1}{(1-x)^2}$$

$$f''(x) = \frac{2}{(1-x)^3}$$

$$f'''(x) = \frac{6}{(1-x)^4}$$

$$f^{(4)}(x) = \frac{24}{(1-x)^5}$$

$$f(0) = 1, f'(0) = 1, f''(0) = 2, f'''(0) = 6, f^{(4)}(0) = 24$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \dots$$